Ampere Circuital Law

Objectives

After going through this lesson, the learners will be able to:

- Interpret Ampere Circuital Law and its proof
- Apply of Ampere Circuital Law to obtain
  - Magnetic field due to a long-straight wire
  - Magnetic field due to a current carrying long-straight solenoid
  - Magnetic field of a current carrying toroidal (endless) solenoid
- Identify the direction and the magnitude of a Force on a charge moving in a Magnetic Field.
- Describe Lorentz Force and define Unit of Magnetic Field

Content Outline

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- Application of Ampere Circuital Law
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  - Magnetic field of long-straight solenoid
  - Magnetic field of a toroidal (endless) solenoid
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Unit Syllabus

Unit –III: Magnetic Effects of Current and Magnetism-10 Modules

Chapter-4: Moving Charges and Magnetism

Concept of magnetic field; Oersted's experiment.

Biot - Savart law and its application to the current carrying circular loop.

Ampere's law and its applications to infinitely long straight wire; Straight and toroidal solenoids, Force on a moving charge in uniform magnetic and electric fields; Cyclotron.

Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter.

Chapter-5: Magnetism and Matter

Current loop as a magnetic dipole and its magnetic dipole moment. Magnetic dipole moment of a revolving electron. Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements.

Para-, dia- and ferro - magnetic substances, with examples; Electromagnets and factors affecting their strengths; Permanent magnets.

Module Wise Distribution of Unit Syllabus

Module 1	• Introducing moving charges and magnetism • Direction of magnetic field produced by a moving charge • Concept of Magnetic field • Oersted's Experiment • Strength of the magnetic field at a point due to current carrying conductor • Biot-Savart Law • Comparison of coulomb's law and Biot Savart's law
Module 2	• Applications of Biot- Savart Law to current carrying circular loop, straight wire • Magnetic field due to a straight conductor of finite size

	● Examples
Module 3	● Ampere’s Law and its proof ● Application of Ampere's circuital law: straight wire, straight and toroidal solenoids. ● Force on a moving charge in a magnetic field ● Unit of magnetic field ● Examples
Module 4	● Force on moving charges in uniform magnetic field and uniform electric field. ● Lorentz force ● Cyclotron
Module 5	● Force on a current carrying conductor in uniform magnetic field ● Force between two parallel current carrying conductors ● Definition of ampere
Module 6	● Torque experienced by a current rectangular loop in uniform magnetic field ● Direction of torque acting on current carrying rectangular loop in uniform magnetic field ● Orientation of a rectangular current carrying loop in a uniform magnetic field for maximum and minimum potential energy
Module 7	● Moving coil Galvanometer- ● Need for radial pole pieces to create a uniform magnetic field ● Establish a relation between deflection in the galvanometer and the current its current sensitivity ● Voltage sensitivity ● conversion to ammeter and voltmeter ● Examples
Module 8	● Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. ● Torque on a magnetic dipole in a uniform magnetic field. ● Explanation of magnetic property of materials

Module 9	• Dia, Para and Ferro-magnetic substances with examples. Electromagnets and factors affecting their strengths, permanent magnets.
Module 10	• Earth's magnetic field and magnetic elements.

Module 3

Words You Must Know

- **Coulomb's law:** The force of attraction or repulsion between two point charges is directly proportional to the product of two charges (q_1 and q_2) and inversely proportional to the square of the distance between them. It acts along the line joining them.
- **Electric current:** The rate of flow of charge with time.
- **Magnetic field lines:** It is a curve, the tangent to which a point gives the direction of the magnetic field at that point.
- **Maxwell's corkscrew rule or right hand screw rule:** It states that if the forward motion of an imaginary right handed screw is in the direction of the current through a linear conductor, then the direction of rotation of the screw gives the direction of the magnetic lines of force around the conductor.
- **Biot-Savart law:** According to Biot-Savart law, the magnetic field *dB* at *P* due to the current element *idl* is given by

$$dB = \frac{\mu_0 Idl\, sin\theta}{r^2}$$

- **Right hand thumb rule or curl rule:** If a current carrying conductor is imagined to be held in the right hand such that the thumb points in the direction of the current, then the tips of the fingers encircling the conductor will give the direction of the magnetic lines of force.

Introduction

We discussed two approaches for calculating the electric field Coulomb's law and Gauss's law. There are also several ways to calculate the magnetic field produced by a current.

One approach is the Biot Savart's Law, which similar to Coulomb's law, treats each small piece of wire as a separate source of B, and is mathematically complicated. Another approach, which was proposed by Ampere, and known as Ampere's circuital law, is very useful when magnetic field lines have a simple symmetry.

This is similar to Gauss's law in electrostatics for determining the electric fields, which is most useful when the electric field is highly symmetrical.

Ampere's circuital law is an alternative and appealing way in which the Biot-Savart law can be expressed. The purpose of the law is to find the value of the magnetic field at a location from a current carrying conductor, if the magnetic field is symmetrical.

Some examples of symmetrical fields are those around a straight conductor and a circular loop with steady current.

Ampere's Circuital law

This law given by Ampere, provides us with **an alternative way of calculating the magnetic field due to a given current distribution.**

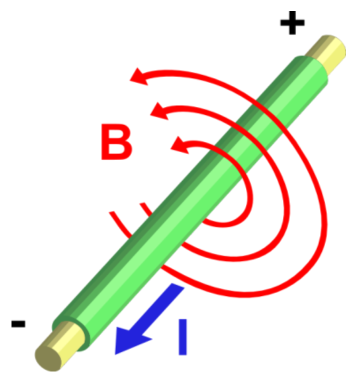
As said before, this law is in a way similar to Gauss's law in electrostatics, which again provides us with an alternative way of calculating the electric field due to a given charge distribution.

Ampere's circuital law can be written as:

The line integral of the magnetic field around some closed loop is equal to μ_0 times the algebraic sum of the currents which pass through the loop.

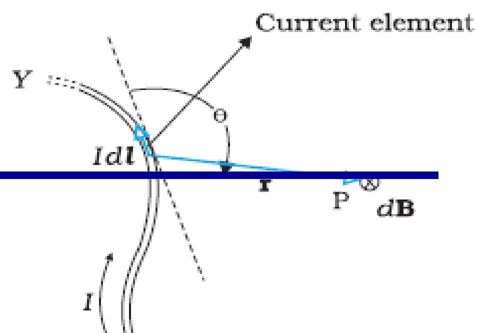
So let us attempt to understand what is meant by:

- line integral
- closed loop
- algebraic sum of currents



The figure shows the magnetic field around a current carrying conductor. From our previous knowledge we take the conventional direction of current (from +ve to –ve), the red concentric circles represent the magnetic field in a plane perpendicular to the wire

From Biot Savart's Law, B is inversely proportional to the square of the distance from the source to the point



of interest.

The source of the field is a vector given by Idl or

the magnetic field is produced by a vector source **I dl.**

The magnetic field is perpendicular to the plane

containing the displacement vector **r** and the current

element ***I* d*l***.

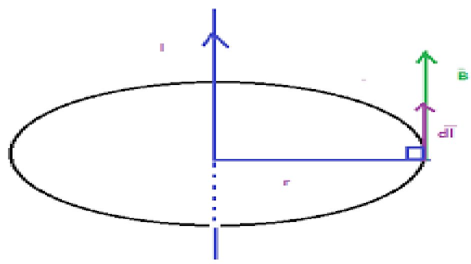
There is angle dependence in the Biot-Savart law

which is not present in the electrostatic. Consider the

diagram.The magnetic field at any point in the direction of d*l* (the dashed line) is zero.

Along this line,

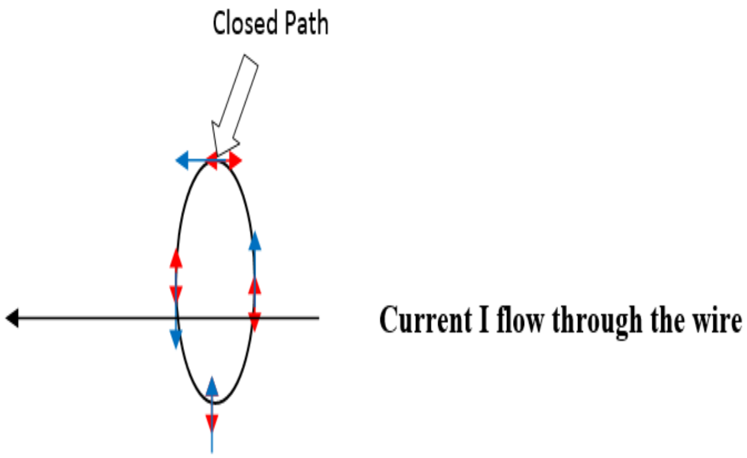
θ = 0, sin θ = 0 and therefore, |dB| = 0.



Amperes' circuital law makes the calculation of B easier in many cases:

Consider a long straight current carrying wire encircled by magnetic field lines and imagine travelling around a closed path that also encircles the wire

Ampere's law relates the magnetic field along the path to the electric current enclosed by this



path

Let us travel along the path taking steps of length Δl and let B be the component of the magnetic field parallel to these steps.

According to ampere's law over the entire closed loop which we have taken as a circular loop

$\sum B.\Delta l = \mu_0\ I$

In simple terms law relates the magnetic field on the perimeter of the region to the current that passes through the region.

How big should our step be to estimate B easily? it should be as small as possible.

So,Let each such step be an element of length dl. This is a vector as we would move clockwise or anticlockwise.

We take the value of the tangential component of the magnetic field, B_T at this element and multiply it by the length of that element dl. B_T is also a vector; hence it is a product of two vectors.

The right hand side of the equation is a scalar- product of permeability and magnitude of current

Note

B_T . dl = B dl cos θ = B_T dl as θ = 0

All such products are added together.

We consider the limit as the lengths of elements get smaller and smaller making their number larger. The sum then tends to an integral.

Ampere’s law states that this integral is equal to μ_0 times the total current.

$\int B.dl = \mu_0\ I$

It is often written as line integral of B.dl

$\oint B.dl = \mu_0 I$

Where *I* is the total current through the boundary of a surface. The integral is taken over the closed loop.

The relation above involves a sign-convention, given by the right-hand rule.

Let the fingers of the right-hand be curled in the sense the boundary is traversed in the loop integral B.*dl*. Then the direction of the thumb gives the sense in which the current *I* is regarded as positive.

In general, for several applications, a much simplified version of the above equation proves sufficient. We shall assume that, in such cases, it is possible to choose the loop (called an ***amperian loop***) such that at each point of the loop, either

i. B is tangential to the loop and is a non-zero *constant* B,

ii. B is normal to the loop, *or*

iii. B vanishes.

Now, let L be the length of the loop for which B is tangential.

Let I be the current enclosed by the loop.

Then, the above equation reduces to,

$$\boldsymbol{BL = \mu_0 I}$$

When there is a system with symmetry such as that for a *straight infinite current-carrying wire* Ampere's law enables an easy evaluation of the magnetic field, much the same way Gauss' law helps in determination of the electric field.

The boundary of the loop chosen is a circle and the magnetic field is tangential to the circumference of the circle.

The boundary can be of any shape but its perimeter is not easily predictable as in the case of a circular boundary where it is the circumference of the circle = $2\pi r$

The law gives, for the left hand side B. $2\pi r$

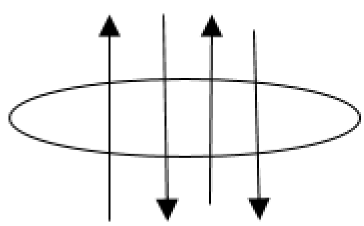
We find that the magnetic field at a distance r outside the wire is *tangential* and given by

$$B.\ 2\pi r = \mu_0 I$$

$$B = \mu_0 I/2\pi r$$

So far we have taken care of what is meant by closed loop, line integral and why a circular amperian loop is advantageous for calculation of perimeter of the closed loop.

Now what is meant by the algebraic sum of currents?



The amperian loop is not equidistant from ,In the figure the amperian loop has four straight conductors say each with a current of 2 A but in different directions net algebraic sum of currents = 0 while total current = 4 × 2 A

So the magnetic field at any point on the loop will be 0.

Think About This

- All the conductors as in the earlier case but the field is zero?
- If the conductors were at different angles with each other, can we still apply ampere's law?
- Why is symmetry important for simple application of ampere's circuital law?

Ambiguities and Sign Conventions

There are a number of ambiguities in the above definitions that require clarification and a choice of convention.

Three of these terms are associated with sign ambiguities:

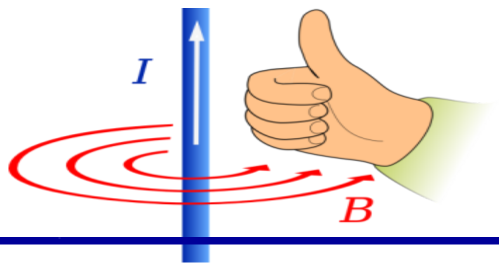
- The line integral $\oint_C$ could go around the loop in either direction (clockwise or counterclockwise);
- The vector Δl or dl could point in either of the two directions along the loop
- I_{enc} is the net current passing through the loop, meaning the current passing through in one direction, minus the current in the other direction—but either direction could be chosen as positive.

These ambiguities are resolved by the right hand grip rule as discussed earlier.

Right Hand Grip Rule

Grasp the wire in your right hand with your extended thumb pointing in the direction of the current. Your fingers will curl around in the direction of the magnetic field.

So how do we choose an amperian loop?



There are infinitely many possible surfaces S that have a boundary curve C as their border making up the loop.

Imagine a soap film on a wire loop, which can be deformed by moving the wire.

Which of those surfaces is to be chosen?

If the loop does not lie in a single plane, for example, there is no one obvious choice.

The answer is that it does not matter; it can be proved that any surface with boundary C can be chosen. But the calculation of the perimeter of the boundary will not be obvious.

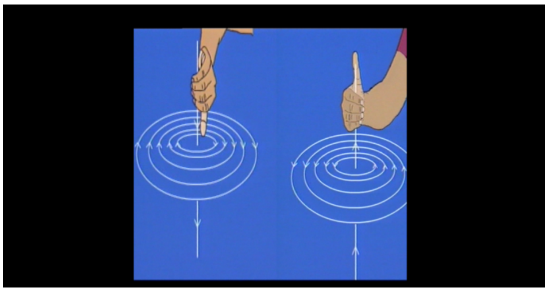
Analysis of result of B due to a Current Carrying Conductor of infinite length

$B = \mu_0 I/2\pi r$.

The above result for the infinite wire is interesting from several points of view.

(i) It implies that the field at every point on a circle of radius r, (with the wire along the axis), is the same in magnitude. In other words, the magnetic field possesses what is called a *cylindrical symmetry*. The field that normally can depend on three coordinates depends only on one: r. Whenever there is symmetry, the solutions simplify

(ii) The field direction at any point on this circle is tangential to it. Thus, the lines of constant magnitude of magnetic field form concentric circles.



Notice now the iron filings form concentric circles.

These lines called *magnetic field lines* form closed loops. This is unlike the electrostatic field lines which originate from positive charges and end at negative charges. The expression for the magnetic field of a straight wire provides a theoretical justification to Oersted's experiments.

(iii) Another interesting point to note is that even though the wire is infinite, the field due to it at a nonzero distance is *not* infinite. It tends to blow up only when we come very close to the wire.

(iv) The field is directly proportional to the current and inversely proportional to the distance from the (infinitely long) current source. There exists a simple rule to determine the direction of the magnetic field due to a long wire. This rule, called the *right-hand rule**, is: *Grasp the wire in your right hand with your extended thumb pointing in the direction of the current. Your fingers will curl around in the direction of the magnetic field.*

(v) Ampere's circuital law is not new in content from Biot-Savart law, both relate the magnetic field and the current, and both express the same physical consequences of a steady electrical current.

Ampere's law is to Biot-Savart law, what Gauss's law is to Coulomb's law. Both, Ampere's and Gauss's laws relate a physical quantity on the periphery or boundary (magnetic or electric field) to another physical quantity, namely, the source, current or charge

We also note that Ampere's circuital law holds for steady currents which do not fluctuate with time.

Ampere's Circuital Law Facts

- Ampere's circuital law in magnetism is analogous to Gauss's law in electrostatics.
- This law is also used to calculate the magnetic field due to any given current distribution

  This law states that: "**The line integral of resultant magnetic field along a closed plane curve is equal to μ_0 time the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant**"

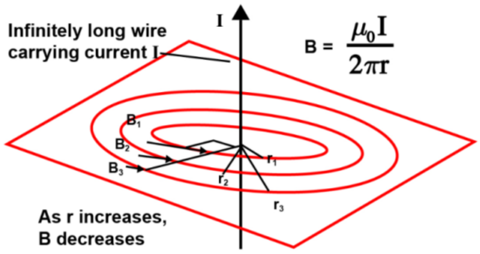
  Thus

$$\oint B.dl = \mu_0 I_{enc}$$

where

μ_0 is the permeability of free space and,

I_{enc} is the net current enclosed by the loop as shown below in the figure



- The symbol "$\oint B. dl$" means that scalar product B.dl is to be integrated around the

M closed loop known as Amperian loop whose beginning and end point are same.

- Anticlockwise direction of integration as chosen in the above figure is an arbitrary one. We can also use clockwise direction of integration for our calculation depending on our convenience.
- To apply the ampere's law we divide the loop into infinitesimal segments dl and for each segment, we then calculate the scalar product of B and dl.
- B in general varies from point to point so we must use B at each location of dl
- Amperian Loop is usually an imaginary loop or curve, which is constructed to permit the application of ampere's law to a specific situation.

Ampere's law, as stated above, provides us with an easy, quick and convenient way of calculating the line integral of the magnetic field over a given closed path or loop. We only need to know, or calculate, the (net) current enclosed by that loop.

However we need to know the magnetic field itself, rather than just the value of its 'line integral'. Therefore, we need a special closed path for which one can calculate the magnetic field, rather than just its line integral using the circuital law. The choice of such a (special) closed path becomes possible only for a limited range of current distributions that have some sort of symmetry or 'idealization' associated with them.
Ampere's circuital law, therefore, is a handy tool for calculating the magnetic field only for a (very much) limited range of current distributions. This 'built in' limitation of this law restricts its use in practical situations.

Proof of Ampere's Law:

Consider a long thin wire carrying a steady current I. Suppose that the wire is oriented such that the current flows along the positive z-axis. Consider some closed loop 'C' in the x-y plane which circles the wire in an anti-clockwise direction, looking down the z-axis. Suppose that dl is a short straight-line element of this loop. Let us form the dot product of this element with the local magnetic field B. Thus,

B. dl=B dl cosθ

where θ is the angle subtended between the direction of the line element and the direction of the local magnetic field.

We can calculate a 'dw' for every line element which makes up the loop 'C'. If we sum all of the 'dw' values thus obtained, and take the limit as the number of elements goes to infinity, we obtain the *line integral*

$dw = B.dl = B.dl cos\theta$

$w = \oint B.dl$ What is the value of this integral?

let us consider a special case.

Suppose that 'C' is a circle of radius r centered on the wire.

In this case, the magnetic field-strength is the same at all points on the loop.

In fact, from Biot Savart's law,

$$B = \frac{\mu_0 I}{2\pi r}$$

Moreover, the field is everywhere parallel to the line elements which make up the loop.

Thus,

$$w = 2\pi r\, B = \mu_0 I$$

or

$$\oint B.dl = \mu_0 I$$

In other words, the line integral of the magnetic field around some circular loop C, centered on a current carrying wire, and in the plane perpendicular to the wire, is equal to μ_0 times the current flowing in the wire.

Note that this answer is independent of the radius r of the loop: *i.e.*, the same result is obtained by taking the line integral around *any* circular loop centered on the wire.

In 1826, Ampere demonstrated that Equation

$$\oint B.dl = \mu_0 I$$

This equation holds for *any* closed loop which circles around *any* distribution of currents (need not imagine the currents to be at its centre). Thus, Ampere's circuital law can be written as:

The line integral of the magnetic field around some closed loop is equal to the μ_0 times the algebraic sum of the currents which pass through the loop.

Ampere's circuital law is to magnetostatics (the study of the magnetic fields generated by steady currents) whileGauss' law is to electrostatics (the study of the electric fields generated by stationary charges).

Like Gauss's law, Ampere's circuital law is particularly useful in situations which possess a high degree of symmetry.

Magnetic Field Inside a Long Straight Wire Carrying Steady Current

All the above discussion was for magnetic field at a distance away from the center of the wire, obviously outside the wire.But is there a magnetic field within the wire? We can use Ampere's law to determine the magnetic field inside the wire.

Assuming

- The current must be through a wire of uniform cross section.
- The current density should be uniform.
- The material of the straight wire should be homogeneous with the same resistivity.

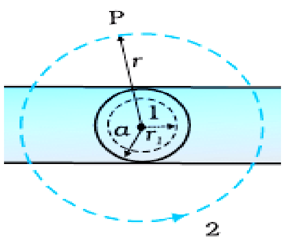
For this let us imagine a conductor wire through which a current I flows so we have a long straight wire of a circular cross-section (radius *a*) carrying steady current *I*.

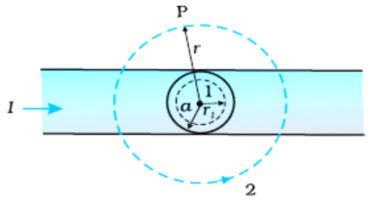
The current *I* is uniformly distributed across this cross-section.

Let us calculate the magnetic field in the region

i) $r > a$ and

ii) $r < a$





i. Consider the case $r > a$. The Amperian loop, labeled 2, is a circle concentric with the cross-section. For this loop,

$L = 2\,\pi\, r$

I_e = Current enclosed by the loop = I

The result is the familiar expression for a long straight wire

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B \propto \frac{1}{r}$$

ii. Consider the case $r < a$. The Amperian loop is a circle labeled 1. For this loop, taking the radius of the circle to be r,

Now the current enclosed I_e is not I, but is less than this value.

Since the current distribution is uniform, the current enclosed is,

$$I_e = I\left(\frac{\pi r^2}{\pi a^2}\right) = \frac{I r^2}{a^2}$$

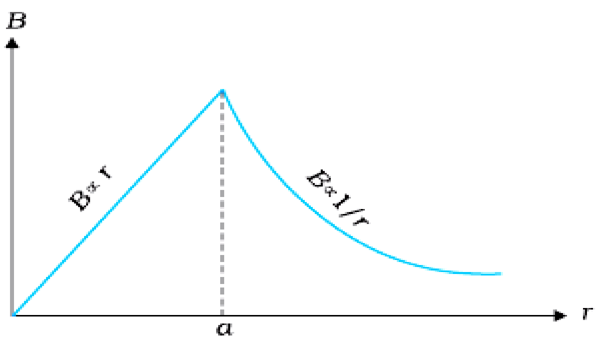
Using Ampères law

$$B(2\pi r) = \mu_0 \frac{I r^2}{a^2}$$

$$B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r$$

$$B \propto r \qquad (r < a)$$

The above result can be represented graphically



Think About This

- What if the wire was not uniform?
- What if the wire was not homogeneous?
- What if we had a collection of thin wires bundled together?
- What if the cross section of the wire is not circular or the wire is not cylindrical?
- What is the value of the magnetic field at the centre of the wire due to current flowing through it?
- Where is the magnetic field maximum?

Applications of Ampere's Circuital Law

We have determined the magnetic field intensity due to current carrying a straight conductor of infinite length by applying Biot-Savart's law. However in symmetric cases it is more convenient to use Ampere's law for determining the magnetic field. Here we shall try Ampere's law to obtain magnetic fields.

i) Magnetic field around a long straight conductor

Consider a straight long conductor carrying steady current I. In order to find the magnetic field at a point P at a distance r from the conductor, consider a circular path containing the point with the conductor as axis.

As the magnetic field at all points on the circular path is tangential and of the same magnitude we get Line integral of B.dl equal to B × (line integral of dl) =B × 2πr

The result is the same as that of the equation.

$$\mathbf{B.\ 2\pi r = \mu_0\ I}$$

$$\boldsymbol{B = \mu_0\ I/2\pi r.}$$

ii) Magnetic field inside a straight conductor carrying steady current

a. At a location at a distance r from the center of the coil r >>a the radius of the wire

$$B = \frac{\mu_0 I}{2\pi r}$$

b. At a location at a distance ‘r’ from the center of the cylindrical wire of radius ‘a’

$$B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r$$

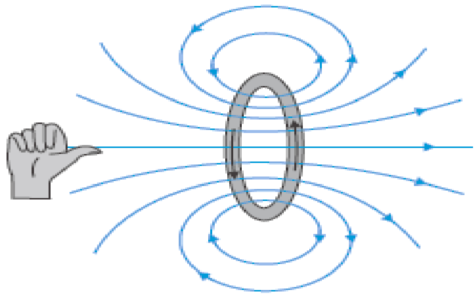
$$B \propto r \;\; (r < a)$$

iii) Magnetic field at the center of a circular coil

It should be noted that while Ampere's circuital law holds for any loop, it **may not always** facilitate an evaluation of the magnetic field in every case.

For example, for the case of the circular loop of radius R, carrying a current = I. As discussed earlier it cannot be applied to extract the simple expression.

In this case, using Ampere's circuital law is very difficult and instead of making our calculations simpler, it makes it really hard.



$$B = \frac{\mu_0 I}{2R}$$

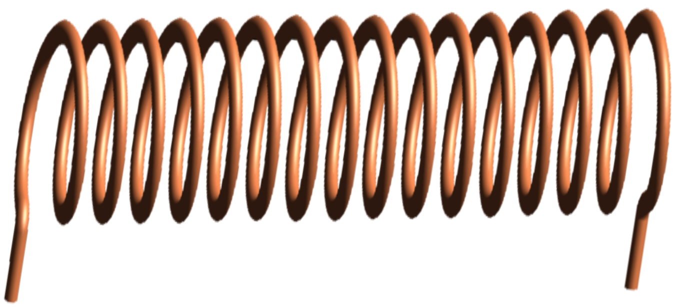
For the magnetic field B at the centre of the loop.

However, there exist a large number of situations of high symmetry where the law can be conveniently applied.

iv) Magnetic field due to a steady current in a solenoid

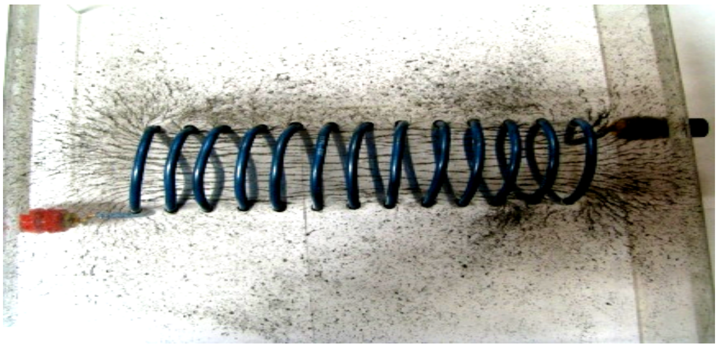
A current carrying uniform wire wound in the form of coil or a helix is called solenoid.

In fact, it is a combination of a number of circular coils spread over a length. Now let us investigate the magnetic field due to a solenoid.

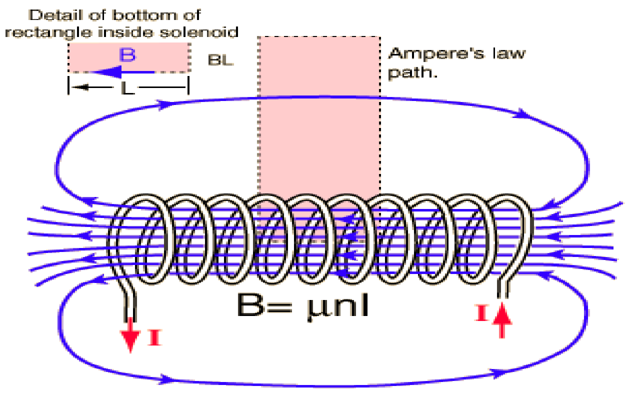


A solenoid of definite length behaves exactly in the same way as that of a bar magnet.

The figure shows this behavioral resemblance with iron filings around a current carrying solenoid



However if we consider an infinitely long solenoid, it has been established experimentally that the magnetic field inside the solenoid is uniform and the magnetic field outside the solenoid vanishes.



Let us now derive the expression for the magnetic field due to the current in the solenoid.

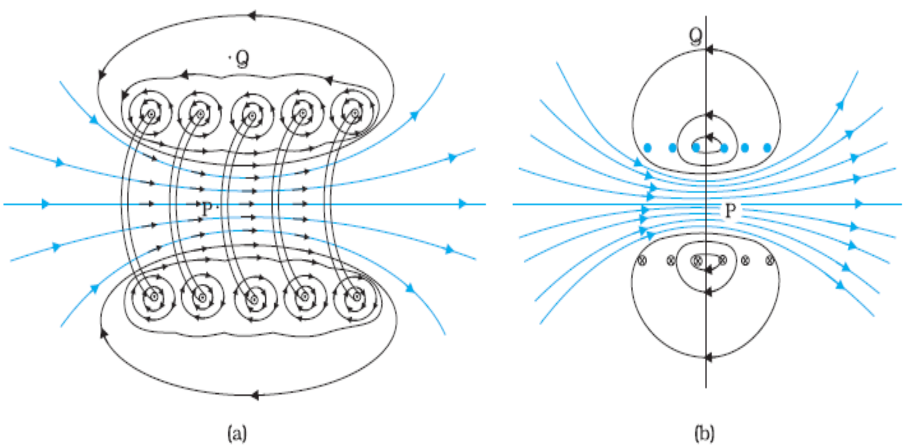
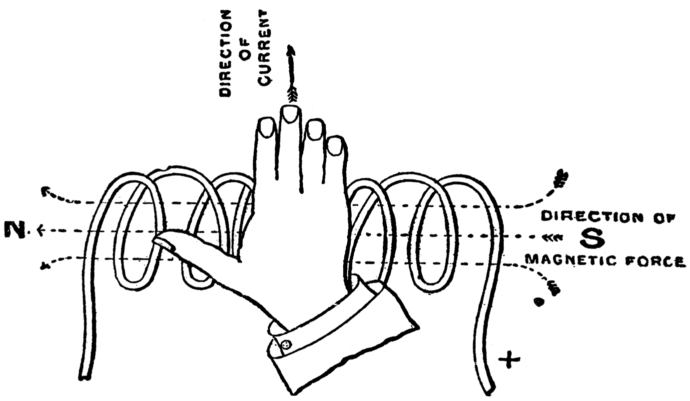


Figure displays the magnetic field lines for a finite solenoid.

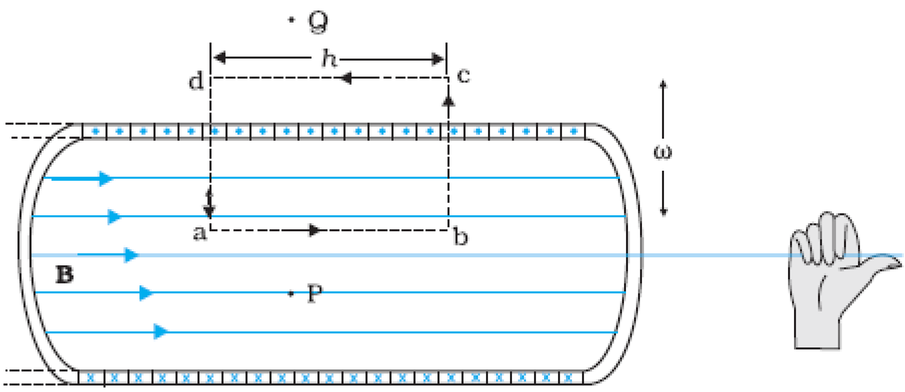
- We show a section of this solenoid in an enlarged manner in Fig. (a).

- Figure (b) shows the entire finite solenoid with its magnetic field.
- In Fig. (a), it is clear from the circular loops that the field between two neighboring turns vanishes.
- In Fig.(b), we see that the field at the interior mid-point P is uniform, strong and along the axis of the solenoid.
- The field at the exterior mid-point Q is along the axis of the solenoid with no perpendicular or normal component.
- This field outside the solenoid is weak and approaches zero.

We shall assume that the field outside is zero. The field inside becomes everywhere parallel to the axis. Direction of field B- from south to north inside the solenoid - given by right hand grip rule



Let us now determine the magnetic field due to an infinitely long solenoid.Consider an infinitely long solenoid of n turns per unit length.We can imagine a rectangular Amperian loop abcd.



Along cd the field is zero as argued above.

For, if you assume that the current in individual loops have no component along the direction of the axis of the loop.

Along transverse sections bc and ad, the field component is zero. Thus, these two sections make no contribution.

Let the field along ab be B. Thus, the relevant length of the Amperian loop is, $L = h$.

If n is the number of turns per unit length, then the total number of turns is $n\,h$.

The enclosed current is, $I_e = I\ (n\ h)$, where I is the current in the solenoid, since current in each loop adds up to total enclosed current within the Amperian loop.

From Ampere's circuital law

$$BL = \mu_0 I$$

$$B\,h = \mu_0 I\,(n\,h)$$

$$B = \mu_0 I\,n$$

The direction of the field is given by the right-hand rule. The solenoid is commonly used to obtain a uniform magnetic field.

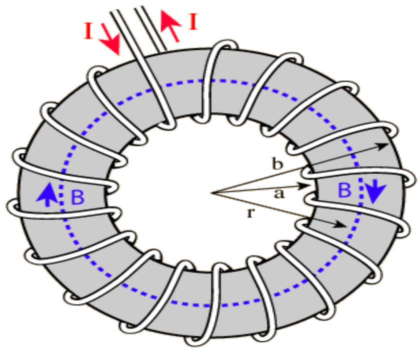
If the core (space inside the solenoid coil) of the solenoid is filled with a medium of relative permeability μ_r, then the above equation is modified as

$$\mathbf{B = \mu_o\,\mu_r\,n\,I}$$

The above expression makes it clear that the magnetic field inside a long solenoid is

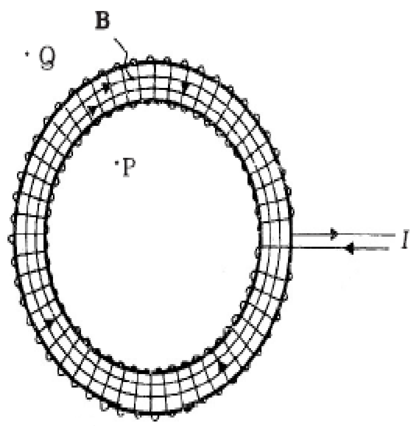
- uniform
- depend on the number of turns per unit length,
- current in the solenoid
- Medium of the core.

v) Magnetic field due to a steady current in a Toroid

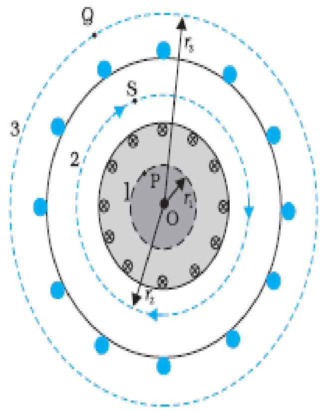


The Toroid is a hollow circular ring on which a large number of turns of a wire are closely wound.

It can be viewed as a solenoid which has been bent into a circular shape to close on itself.



(a)



(b)

Finding the magnetic field inside a Toroid is a good example of the power of Ampere's law, the circuit enclosed by the dashed amperian loop.

It is shown in Fig. (a) carrying a current *I*. We shall see that the magnetic field in the open space inside (point P) and exterior to the Toroid (point Q) is zero. The field B inside the Toroid is constant in magnitude for the *ideal* Toroid of closely wound turns. Fig (b) shows a sectional view of the Toroid.

The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops.

Three circular Amperian loops 1, 2 and 3 are shown by dashed lines.

By symmetry, the magnetic field should be tangential to each of them and constant in magnitude for a given loop.

The circular areas bounded by loops 2 and 3 both cut the Toroid: so that each turn of current carrying wire is cut once by the loop 2 and twice by the loop 3.

Let the magnetic field along loop 1 be B_1 in magnitude.

Then in Ampere's circuital law, $L = 2\pi r_1$

However, the loop encloses no current, so $I_e = 0$. Thus,

$$B_1 (2 \pi r_1) = \mu_0 (0) = 0$$

Thus, the magnetic field at any point P in the open space inside the Toroid is zero.We shall now show that the magnetic field at Q is likewise zero.

Let the magnetic field along loop 3 be B_3.

Once again from Ampere's law $L = 2 \pi r_3$.

However, from the sectional cut, we see that the current coming out of the plane of the paper is cancelled exactly by the current going into it.

Thus,

$$I_e = 0, \text{ and } B_3 = 0.$$

Let the magnetic field inside the solenoid be B. We shall now consider the magnetic field at S. Once again we employ ampere's law. $L = 2 \pi r_2 = 2 \pi r$, where r is the radius of the Toroid

The current is NI

$$B(2 \pi r) = \mu_0 NI B = \frac{\mu_0 NI}{2\pi r}$$

If we compare the two results: for a Toroid and solenoid,

Let r be the average radius of the Toroid and n is the number of turns per unit length.

Then

***N* = 2π*r n* = *(average) perimeter of the Toroid* × *number of turns per unit length* and thus,**

$\boldsymbol{B = \mu_0 n I}$,

i.e., the result for the solenoid!

In an ideal Toroid the coils are circular. In reality the turns of the toroidal coil form a helix and there is always a small magnetic field external to the Toroid.

Solved Examples

Example

A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

Solution

The number of turns per unit length is,

$$n = \frac{500}{0.5} = 1000\ turns\ /m$$

The length l = 0.5 m and radius r = 0.01 m.

Thus, $l/r = 50$ i.e., $l >>> r$

Hence, we can use the *long* solenoid formula, namely,

$$B = \mu_0 n I$$

$$= 4\pi \times 10^{-7}\ Tm/A \times 10^3/m \times 5\ A$$

$$B = 6.28 \times 10^{-3}\ T$$

Example

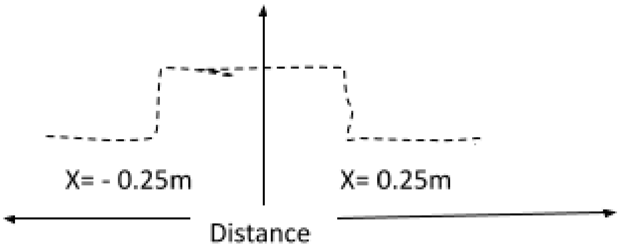
A solenoid coil of 300 turns /m is carrying a current of 5 A. The length of the solenoid is 0.5 m and has a radius of 1 cm. Find the magnitude of the magnetic field inside the solenoid. Draw a graph to show the field strength with distance from the center of the solenoid.

Solution

N= 300 turns /m

I = 5 A

Since it is a long solenoid length much greater than radius



$B = \mu_0 n I$

$= 4\pi \times 10^{-7} Tm/A \times 300/m \times 5A$

$= 1.9 \times 10^{-3}$ T

Example

A wire of radius 0.5 cm carries a current of 100A, which is uniformly distributed over the cross section. Find the magnetic field

a. At 0.1 cm from the axis of the wire
b. At the surface of the wire
c. At a point 0.2 cm from the surface of the wire

Solution

a = 0.5 cm = 0.5 x 10^{-2} m, I = 100 A

a. $B_{inside} = \frac{\mu_0 I}{2\pi a^2} r$

$$= \frac{4\pi\times10^{-7}\times100\times0.1\times10^{-2}}{2\times\pi\times\left(0.5\times10^{-2}\right)^{2}}$$

= 8.0 x 10^{-4} T

b. $B_{\text{surface}} = \frac{\mu_0 I}{2\pi a^2} = \frac{4\pi\times10^{-7}\times100}{2\pi\left(0.5\times10^{-2}\right)^{2}} =$ **8 × 10^{-1} T**

c. **B is to be considered 2 cm from the surface of the wire = 2 cm + 5cm from the centre of the wire**

$$\mathbf{B}_{\textbf{outside}} = \frac{\mu_0 I}{2\pi R}$$

$$= \frac{4\pi\times10^{-7}\times100}{2\pi\times0.7\times10^{-2}}$$

$$= 2.8\times10^{-3}\ \text{T}$$

Force on a Moving Charge in a Magnetic Field

Just as static charges produce an electric field, the currents or moving charges produce (in addition) a magnetic field, denoted by **B**(r), again a vector field. It has several basic properties identical to the electric field.

It is defined at each point in space (and can in addition depend on time). Experimentally, it is found to obey the principle of superposition.

We have so far learnt that a current carrying conductor has a magnetic field around it. The magnitude of the field strength given by B can be calculated using Biot Savart's law and Ampere's circuital law.

We will now understand the force of a moving charge in a magnetic field; this magnetic field may be due to a magnet or a current carrying conductor.

This force was given first by H.A. Lorentz based on the extensive experiments of Ampere and others. It is called the *Lorentz force*.

The origin of the mechanical force can be understood as due to interaction between the external magnetic field and that produced by the moving charge.

You have already studied in detail the force due to the electric field. If we look at the interaction of charges with the magnetic field, we find the following features.

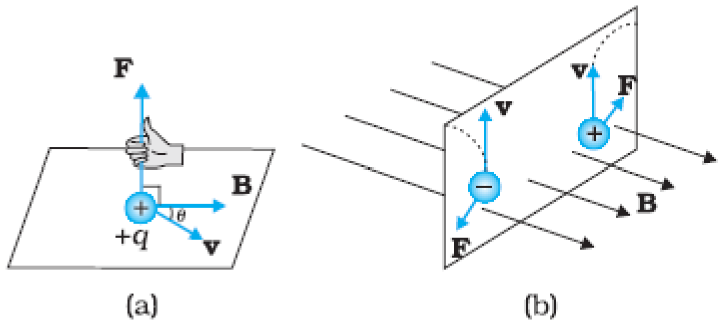
- It depends on q, v and B (charge of the particle, the velocity and the magnetic field respectively).
- Force on a negative charge is opposite to that on a positive charge.
- The magnetic force q [v × B] includes a vector product of velocity and magnetic field.

- The vector product makes the force due to the magnetic field vanish (become zero) if velocity and magnetic field are parallel or antiparallel.
- The force acts in a direction perpendicular to both the velocity and the magnetic field.
- Its direction is given by the screw rule or right hand rule for vector (or cross) product
- If a particle carrying a positive charge *q* and moving with velocity v through a point P in a magnetic field experiences a deflecting force F , then the magnetic field at a point P is defined by the equation

$$F = q(v \times B)$$

- The magnetic force is zero if charge is not moving (as then |v|= 0). Only a moving charge experiences the magnetic force.

Figure shows direction of F, which causes a mechanical force on the moving charged particle making it deflect from its path.



The magnitude of the force on the charged particle is

$$F= q\ v \sin \theta\ B$$

Where 'θ' is the angle between **v** and **B**. The force is maximum when **v** and **B** are at right angles i.e. $\theta = 90^0$. This is given by

$$F_{max} = q\ v\ B$$

If **v** is parallel to **B** (θ = 0° or 180°), then sin θ =0 or F = 0

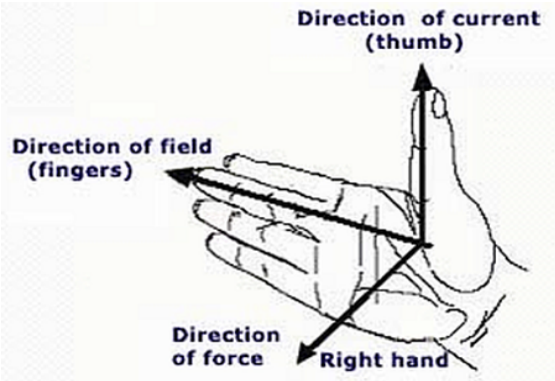
This means that if the charged particle is moving parallel or antiparallel to the magnetic field it does not experience any force. In fact, this defines the direction of the magnetic field.

If a charged particle moving through a point P in a magnetic field does not experience a deflecting force, then we can say that the particle is moving along or anti-parallel to **B.**

Again in the equation, if v = 0, then F = 0. This means that if the charged particle is stationary in the magnetic field, then it does not experience any force. (It is important to note that a 'stationary' charged particle in an electric field does experience a force).

Direction of force on a moving charge in a Magnetic field

Right-hand Palm Rule: Open the right-hand and place it so that tips of the fingers point in the direction of the field and thumb in the direction of velocity of the positive charge then the palm forces towards the force as shown in figure.



Think About This

- Why stationary charges are not affected by external magnetic fields?
- Why is there no effect on a moving charge in the direction of the field?
- Why is there a maximum force when the angle between the magnetic field and velocity vector is 90^0?
- What changes would occur in the value and direction of F, if the magnetic field B is gradually increased or decreased?
- What changes would occur in the magnetic field B if the velocity of the charged particle gradually increased or decreased?
- What changes would occur in the value and direction of F, if the velocity of the charged particle gradually increased or decreased?

Unit of Magnetic Field Intensity B

The expression for the magnetic force helps us to define the unit of the magnetic field, if one takes q, F and v, all to be unity in the force equation $F = q\ [\ v \times B] = q\ v\ B \sin\theta$, where θ is the angle between v and B.

The magnitude of magnetic field B is 1 SI unit, when the force acting on a unit charge (1 C), moving perpendicular to B with a speed 1m/s, is one newton.

Dimensionally, we have $[B] = [F/qv]$ and the unit of B is Newton second / (coulomb metre).

This unit is called *tesla* (T) named after Nikola Tesla (1856 – 1943). Tesla is a rather large unit. A smaller unit (non-SI) called *gauss* ($=10^{-4}$ tesla) is also often used.

The earth's magnetic field is about 3.6×10^{-5} T.

Another SI unit of magnetic field is weber/metre^2 (wb/m^2). Thus, 1T = 1NA $^{-1}$ m $^{-1}$

Some typical Magnetic fields

Physical Situation	Magnitude of B (in tesla)
1. Surface of neutron start	10^8
2. Typical large field in a laboratory	1
3. Near a small bar magnet	10^{-2}
4. On the earth's surface	10^{-5}
5. Human Nerve fiber	10^{-10}
6. Interstellar space	10-^{12}

Summary

- Ampere's circuital law in magnetism is analogous to gauss's law in electrostatics.
- This law is also used to calculate the magnetic field due to any given current distribution.
- This law states that:

"The line integral of resultant magnetic field along a closed plane curve is equal to μ_0 time the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant"

Thus

$$\oint B.dl = \mu_0 I_{enc}$$

where μ_0 is the permeability of free space, I_{enc} is the net current enclosed by the loop.

- The circular sign in equation means that scalar product B.dl is to be integrated around the closed loop known as Amperian loop whose beginning and end point are same.
- We can use any direction of integration for our calculation depending on our convenience.
- To apply the ampere's law we divide the loop into infinitesimal segments **dl** and for each segment, we then calculate the scalar product of **B** and **dl.**
- **B** in general varies from point to point so we must use **B** at each location of dl.
- Amperian Loop is usually an imaginary loop or curve ,which is constructed to permit the application of ampere's law to a specific situation .

- Ampere's law can be used to find:
    - Magnetic field due to a current carrying conductor outside the conductor
    - Magnetic field due to a current carrying conductor inside the conductor
    - Magnetic field due to a current carrying conductor on its surface
    - Magnetic field due to a current carrying conductor in the shape of a solenoid
    - Magnetic field due to a current carrying conductor in the shape of a Toroid
- A mechanical force acts on a moving charge in a magnetic field given by $\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$
- Direction of the force is given by right hand palm rule
- Unit B is Tesla.